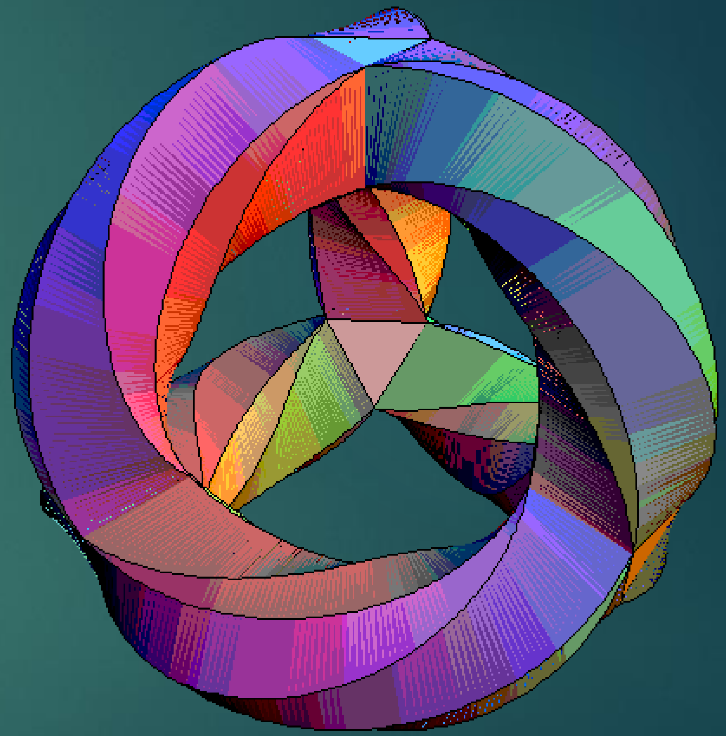
Hunting for Hierarchies in $PSL_2(7)$



MICHAEL JAY PEREZ

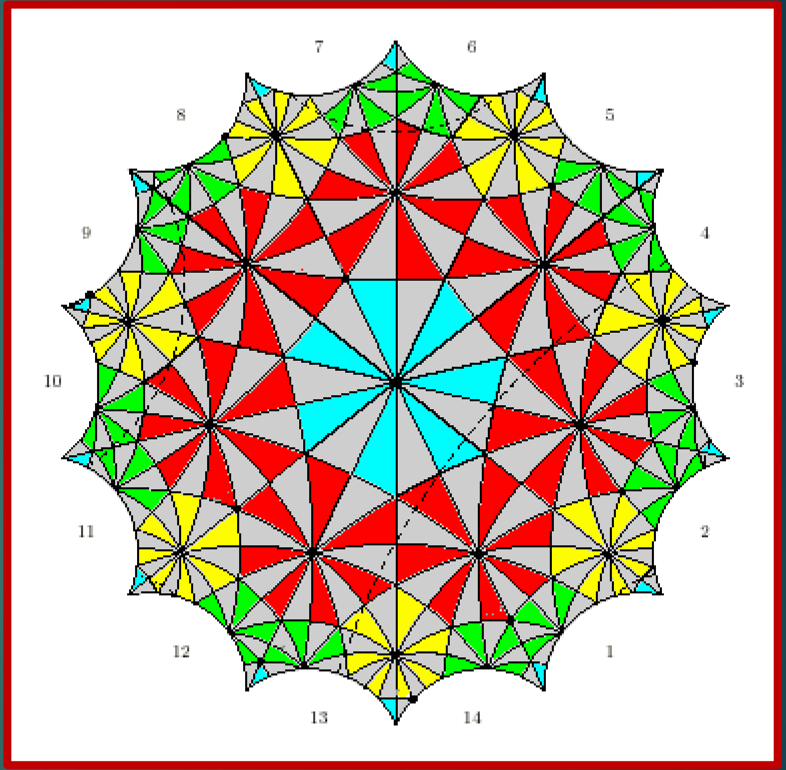
FERMILAB THEORY SEMINAR

FEB 5, 2015

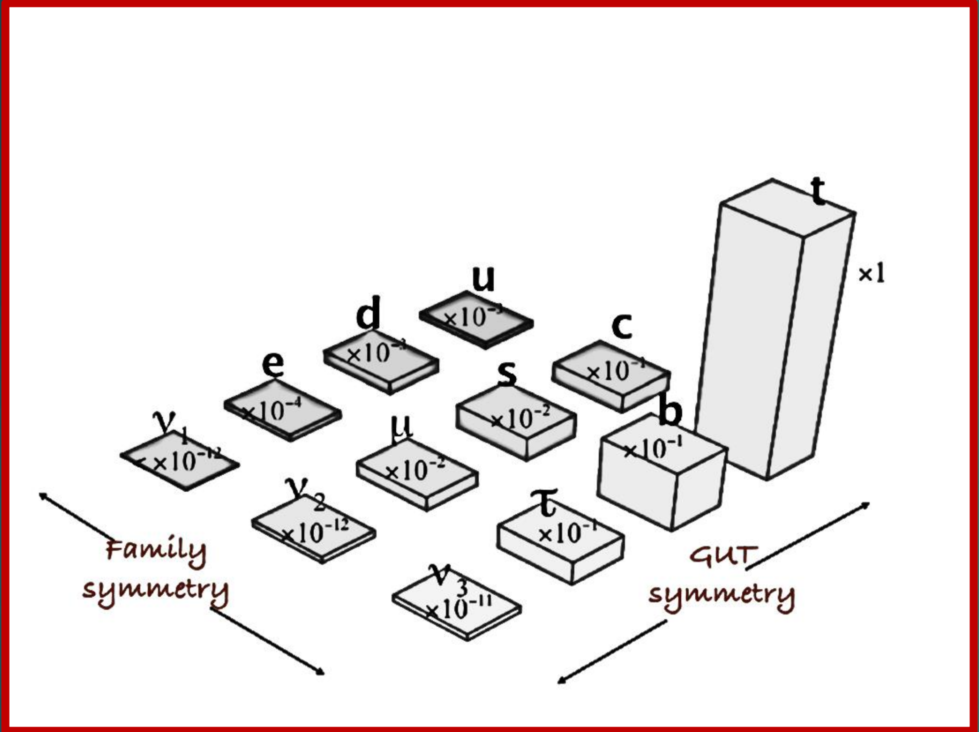
ARXIV : 1412.6107 / 1311.4553

Outline

- A Handful of Hierarchies
  - Flavor Ring as a guide for model building
  - What about $\Delta I_W = 0$?
- A Special Majorana Matrix
  - Building the Majorana Matrix in $Z_7 \times | \; Z_3$
- $PSL_2(7)$ Basics
- The Majorana Matrix in $PSL_2(7)$
- Higgses in $PSL_2(7)$
- The μ - Term
  - The "μ - problem"
  - The μ - Term in $PSL_2(7)$
  - Suitable hierarchy?



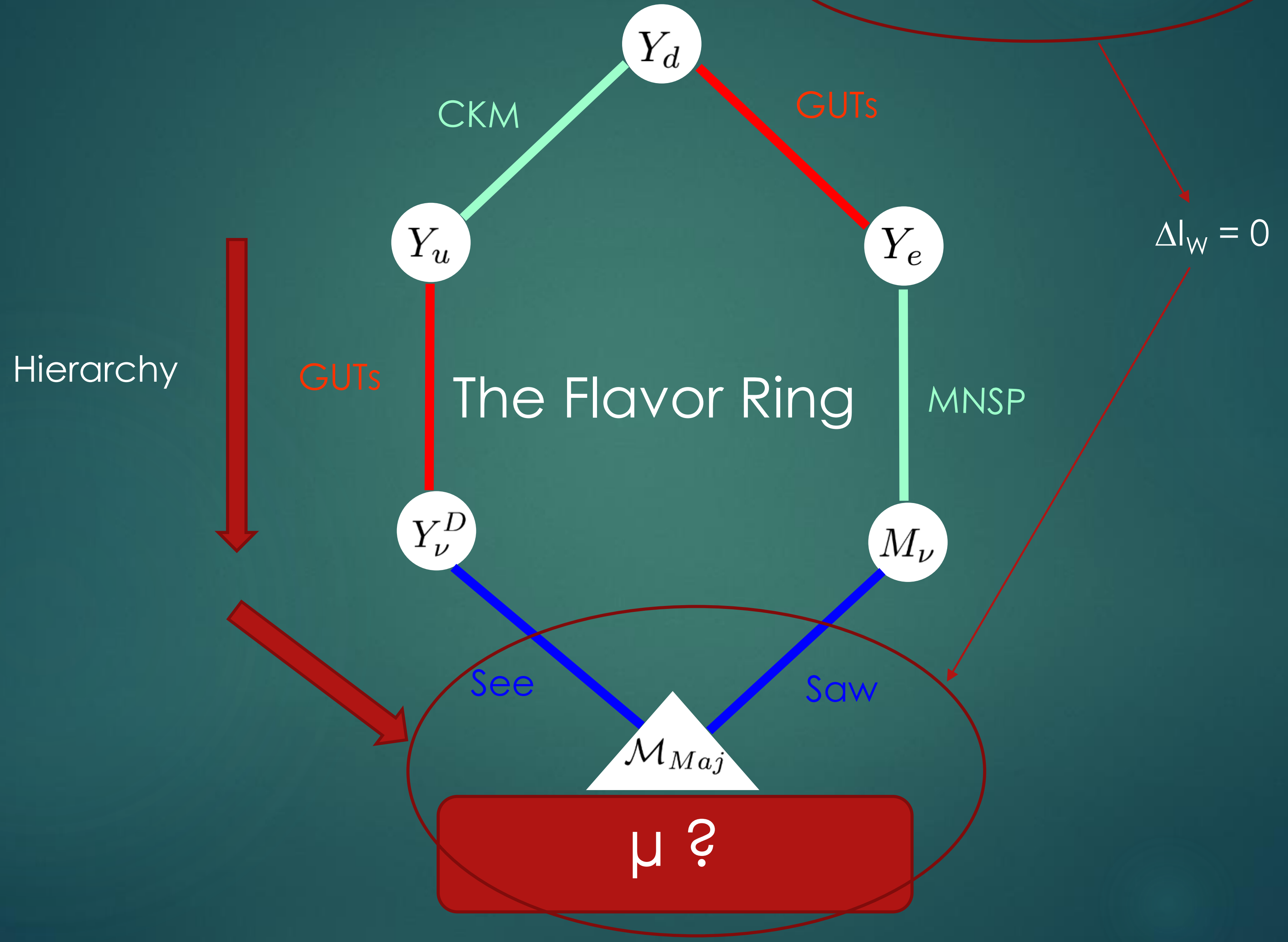
The Flavor Puzzle



GUTs

Family Symmetry

$$\boxed{Y_u\ Q\bar{u}H_u + Y_d\ Q\bar{d}H_d + Y_e\ L\bar{e}H_d + Y_\nu^D\ LNH_u + \mathcal{M}_{Maj}\ N^T N} \boxed{+\mu H_u H_d}$$



Following the Hierarchy

- Up-type quarks display a large hierarchy

$$Y_u \sim y_t \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix} \sim y_t \begin{pmatrix} 10^{-5} & & \\ & 10^{-3} & \\ & & 1 \end{pmatrix}$$

$$\lambda = \sin\theta_c = 0.23$$

- Follow this hierarchy to the Neutrino Sector

$$SO(10): \quad Y_\nu^D = Y_u \sim \begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix}$$

- Then use Family symmetry to follow it to the Higgs Sector

Majorana Sector

- Hierarchy not seen in the light neutrino masses
- Seesaw

$$\boxed{Y_\nu^D\ LNH_u + \mathcal{M}\ N^T N} \Rightarrow \boxed{M_\nu\ \nu_L^T \nu_L}$$

$$\boxed{M_\nu \sim \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}} \Rightarrow \boxed{\mathcal{M} \sim \begin{pmatrix} \sim \lambda^{16} & \sim \lambda^{12} & \sim \lambda^{8} \\ \sim \lambda^{12} & \sim \lambda^{8} & \sim \lambda^{4} \\ \sim \lambda^{8} & \sim \lambda^{4} & \sim 1 \end{pmatrix}}$$

Hierarchical Majorana Matrix

$$\boxed{\mathcal{M} = \begin{pmatrix} a_{11}\,\lambda^{16} & a_{12}\,\lambda^{12} & a_{13}\,\lambda^{8} \\ & a_{22}\,\lambda^{8} & a_{23}\,\lambda^{4} \\ & & a_{33} \end{pmatrix}}$$

- Generic Eigenvalues : $\boxed{1 \;:\; \sim\lambda^{8} \;:\; \sim\lambda^{16}}$

$$\boxed{1 \;:\; \sim 10^{-5} \;:\; \sim 10^{-10}}$$

- Don’t want the lightest eigenvalue to be too light

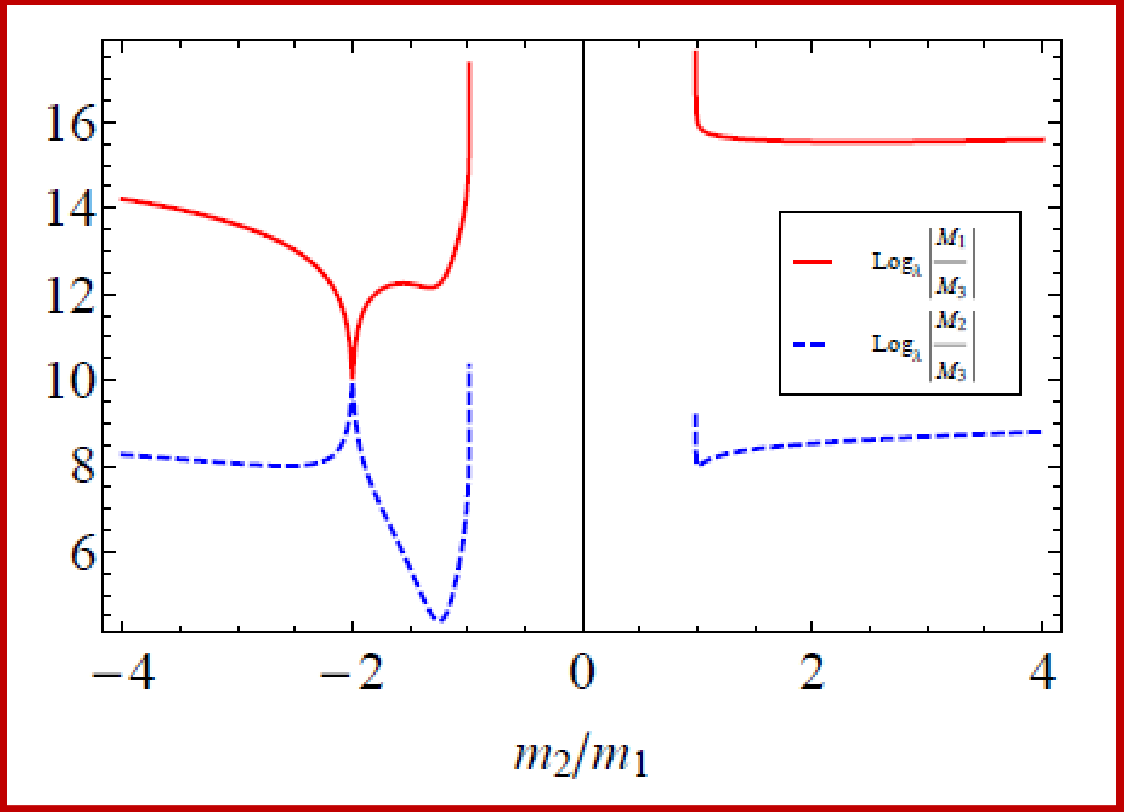
A Special Majorana Matrix (SMM)

- Increased Degeneracy at special point
- (1: $r\lambda^{12}$: $r\lambda^{12}$ ~ 1: 10^{-7} : 10^{-7})
- Gatto – like Relation
- TBM, Normal Hierarchy

$$\tan^2\theta_{12} = -\frac{m_1}{m_2}$$

$$\begin{aligned} m_1 &\approx 0.005\ \text{eV} \\ m_2 &\approx 0.01\ \text{eV} \\ m_3 &\approx 0.05\ \text{eV} \end{aligned}$$

$$\mathcal{M} = \begin{pmatrix} r\lambda^{16} & r\lambda^{12} & r\lambda^{8} \\ r\lambda^{12} & \lambda^{8} & -\lambda^{4} \\ r\lambda^{8} & -\lambda^{4} & 1 \end{pmatrix}, \qquad r = \frac{m_3}{m_1}$$



The SMM

- $M_0 \sim 10^{14}$ GeV

$$\mathcal{M} = M_0 \begin{pmatrix} r\lambda^{16} & r\lambda^{12} & r\lambda^{8} \\ r\lambda^{12} & \lambda^{8} & -\lambda^{4} \\ r\lambda^{8} & -\lambda^{4} & 1 \end{pmatrix}$$

- Precise relations amongst matrix elements
- Can it be natural in a family Symmetry?
- How to produce it ?
  - Tree-Level
  - Higher-Dimensional Operator

Why $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$? $(k_a, g_i) \cdot (k_b, g_j) = ((k_a)^{g_j} k_b, g_i g_j)$

- Smallest Non-Abelian subgroup of SU(3)
- Only 21 Elements
- Irreps : $\mathbf{1}, \mathbf{1}', \bar{\mathbf{1}}', \mathbf{3}, \bar{\mathbf{3}}$

$$\mathbf{3} \otimes \mathbf{3} = (\mathbf{3} + \bar{\mathbf{3}})_s + \bar{\mathbf{3}}_a, \qquad \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} + \mathbf{1}' + \bar{\mathbf{1}}' + \mathbf{3} + \bar{\mathbf{3}}$$

- Cubic Invariants distinguishes diagonal from off-diagonal coupling
- Ex: Top quark mass

$$Q \sim \mathbf{3}, \quad \bar{u} \sim \mathbf{3}, \quad H_u \sim \bar{\mathbf{3}}$$

$$\langle H_u \rangle = v_u \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Q\bar{u}H_u \to v_u \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

SMM in $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$

- An attractive solution with two Familon anti-triplets (N ~ 3)

$$2\sqrt{3}[(NN)_{\bar{\mathbf{3}}}(\bar{\varphi}\,\bar{\varphi}')_{\mathbf{3}} - (NN)_{\mathbf{3}}(\bar{\varphi}\,\bar{\varphi}')_{\bar{\mathbf{3}}}],$$

$$\begin{pmatrix} 2\bar{\varphi}_1\bar{\varphi}'_1 & -(\bar{\varphi}_1\bar{\varphi}'_2 + \bar{\varphi}_2\bar{\varphi}'_1) & -(\bar{\varphi}_1\bar{\varphi}'_3 + \bar{\varphi}_3\bar{\varphi}'_1) \\ & 2\bar{\varphi}_2\bar{\varphi}'_2 & -(\bar{\varphi}_2\bar{\varphi}'_3 + \bar{\varphi}_3\bar{\varphi}'_2) \\ & & 2\bar{\varphi}_3\bar{\varphi}'_3 \end{pmatrix}$$

$$\bar{\varphi} \sim \begin{pmatrix} \bar{\alpha}\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}, \qquad \bar{\varphi}' \sim \begin{pmatrix} \bar{\alpha}'\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}$$

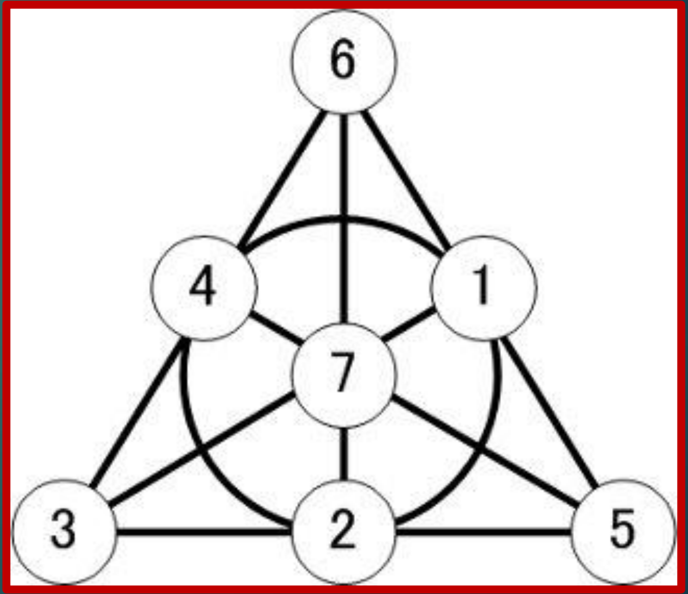
$$\bar{\alpha}\bar{\alpha}' = -\frac{1}{2}(\bar{\alpha} + \bar{\alpha}') = r.$$

- Familon vev's *almost* aligned
- Requires a special linear combination of two operators!
- How do we fix this sign? -> Higher Symmetry!

$PSL_2(7)$ Basics

- Finite subgroup of SU(3) with 168 elements
- Projective Special Linear group of (2 × 2) matrices over F7 (Galois field of seven elements)
- Irreps : 3 , 3^* , 6 , 7 , 8 (6 , 7 , and 8 are all real)

$$\mathcal{PSL}_2(7) \supset \begin{cases} \mathcal{S}_4 \supset \mathcal{A}_4 \\ \\ \mathcal{Z}_7 \rtimes \mathcal{Z}_3 \end{cases}$$



More $PSL_2(7)$

- Maximal discrete subgroup of SU(3)

$$SU(3) \supset \mathcal{PSL}_2(7)$$

$$\begin{array}{lccc}
(10) & \mathbf{3} & = & \mathbf{3} \\
(01) & \overline{\mathbf{3}} & = & \overline{\mathbf{3}} \\
(20) & \mathbf{6} & = & \mathbf{6} \\
(02) & \overline{\mathbf{6}} & = & \mathbf{6} \\
(11) & \mathbf{8} & = & \mathbf{8} \\
(30) & \mathbf{10} & = & \overline{\mathbf{3}}+\mathbf{7} \\
(21) & \mathbf{15} & = & \mathbf{7}+\mathbf{8} \\
(40) & \mathbf{15'} & = & \mathbf{1}+\mathbf{6}+\mathbf{8}
\end{array}$$

$$< a, b \,|\, a^2 = b^3 = (ab)^7 = [a,b]^4 = 1 >, \quad [a,b] = a^{-1}b^{-1}ab$$

$$a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad b = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix},$$

$$\begin{aligned}
\mathbf{3} \otimes \mathbf{3} &= \overline{\mathbf{3}}_a + \mathbf{6}_s \\
\mathbf{3} \otimes \overline{\mathbf{3}} &= \mathbf{1} + \mathbf{8} \\
\mathbf{3} \otimes \mathbf{6} &= \overline{\mathbf{3}} + \mathbf{7} + \mathbf{8} \\
\mathbf{6} \otimes \mathbf{6} &= (\mathbf{1} + \mathbf{6} + \mathbf{6} + \mathbf{8})_s + (\mathbf{7} + \mathbf{8})_a
\end{aligned}$$

Looking up

- Triplet and anti-triplet the same
- Singlet of Z7 X Z3 in the septet
- Six in symmetric product

$$\begin{array}{ccccccc} 3 & \otimes & 3 & = & \bar{3}_a & + & 6_s \\ 3 & \otimes & \bar{3} & = & 1 & + & 8 \end{array}$$

$$\mathcal{PSL}_2(7) \supset \mathcal{Z}_7 \rtimes \mathcal{Z}_3$$

$$\begin{array}{ccl} \mathbf{3} & = & 3 \\ \bar{\mathbf{3}} & = & \bar{3} \\ \mathbf{6} & = & 3 + \bar{3} \\ \mathbf{7} & = & 1 + 3 + \bar{3} \\ \mathbf{8} & = & 1' + \bar{1}' + 3 + \bar{3} \end{array}$$

SMM in $PSL_2(7)$

- Want to produce the linear combination :

$$\boxed{2\sqrt{3}[(NN)_{\bar{\mathbf{3}}}(\bar{\varphi}\,\bar{\varphi}')_{\mathbf{3}} - (NN)_{\mathbf{3}}(\bar{\varphi}\,\bar{\varphi}')_{\bar{\mathbf{3}}}],}$$

- Transformation of Fields :

$$\boxed{N \sim \mathbf{3} \qquad \bar{\varphi} \sim \bar{\mathbf{3}}}$$

$$\boxed{[NN]_{\mathbf{6}} + [NN]_{\bar{\mathbf{3}}}} \qquad \boxed{[\bar{\varphi}\bar{\varphi}']_{\mathbf{6}} + [\bar{\varphi}\bar{\varphi}']_{\mathbf{3}}}$$

$$\boxed{\mathbf{6} \;=\; \mathbf{3} + \bar{\mathbf{3}}}$$

Septet to the rescue

- $PSL_2(7)$ Kronecker Product :

$$\bar{\mathbf{3}} \otimes \mathbf{6} = \mathbf{3} + \mathbf{7} + \mathbf{8}$$

$$\mathbf{6} \otimes \mathbf{6} = (\mathbf{1} + \mathbf{6} + \mathbf{6} + \mathbf{8})_s + (\mathbf{7} + \mathbf{8})_a$$

- Two Z7 X Z3 singlets in product of 6's

$$[NN]_{\mathbf{6}}\,[\bar{\varphi}\bar{\varphi}']_{\mathbf{6}}$$

$$((NN)_{\mathbf{3}}(\bar{\varphi}\bar{\varphi}')_{\bar{\mathbf{3}}_+})_{\mathbf{1}} + ((NN)_{\bar{\mathbf{3}}_+}(\bar{\varphi}\bar{\varphi}')_{\mathbf{3}})_{\mathbf{1}}$$

$$((NN)_{\mathbf{3}}(\bar{\varphi}\bar{\varphi}')_{\bar{\mathbf{3}}_+})_{\mathbf{1}} - ((NN)_{\bar{\mathbf{3}}_+}(\bar{\varphi}\bar{\varphi}')_{\mathbf{3}})_{\mathbf{1}}$$

Simple Underlying Theory

- Coupling we want :

$$\boxed{[NN]_{\mathbf{6}}\,[\bar{\varphi}\bar{\varphi}']_{\mathbf{6}}\,S_{\mathbf{7}}}$$

- Add additional intermediate field which are $PSL_2(7)$ 6's

$$\boxed{[NN]_{\mathbf{6}}\Phi + [\bar{\varphi}\bar{\varphi}']_{\mathbf{6}}\Phi' + \Phi\Phi' S_{\mathbf{7}}}$$

- VEV of S_7 breaks $PSL_2(7)$ down to Z7 X Z3

- Integrating over the 6's gives desired coupling!

Higgs Sector

- Exploring idea of giving Higgs quantum numbers
- How do Higgs fields look in $PSL_2(7)$?
- In previous work Q , U, D, L , E , N ~ 3
- Possible Yukawa Couplings :

$$\mathbf{3} \otimes \mathbf{3} = \overline{\mathbf{3}}_a + \mathbf{6}_s$$

$$\mathrm{Q}\bar{\mathrm{u}}\mathcal{H}_{\mathbf{6}u}, \qquad \mathrm{Q}\bar{\mathrm{d}}\mathcal{H}_{\mathbf{6}d}, \qquad \mathrm{Q}\bar{\mathrm{u}}\mathcal{H}_{\bar{\mathbf{3}}u}, \qquad \mathrm{Q}\bar{\mathrm{d}}\mathcal{H}_{\bar{\mathbf{3}}d},$$

$$[\mathcal{H}_u]_{\mathbf{6}} = (H_u)_{\mathbf{3}} \oplus (\bar{H}_u)_{\bar{\mathbf{3}}}$$

Higgses in $PSL_2(7)$

- Choose the Higgses to be 6's

- Recall : Z7 X Z3

$$Q \sim \mathbf{3}, \quad \bar{u} \sim \mathbf{3}, \quad H_u \sim \bar{\mathbf{3}}$$

$$\langle H_u \rangle = v_u \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Q\bar{u}H_u \to v_u \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

$$\mathbf{6} = \mathbf{3} + \bar{\mathbf{3}}$$

- In our new $PSL_2(7)$ notation, want :

$$\bar{H}_{u1} = v_u$$

Higgs Hierarchies

- Lots of Higgses ! (Family Partners)
- Need a hierarchy between them (only one seen so far!)
- One should be “light”
- Can we use the same Familons to produce the hierarchy ?
- What determines the masses of the Higgs Fields ?

The μ Term : Review

- Minimal Supersymmetric Standard Model has one new parameter μ

$$\boxed{W_{MSSM} = W_{Yukawa} + \mu H_u H_d}$$

- Supersymmetric mass term
  - Gives common mass to H_u and H_d
  - Gives mass to Higgsinos
  - Cubic Scalar couplings
  - Also contributes to chargino and neutralino mass matrices

- This common mass is then split by SUSY breaking terms
- Breaks PQ symmetry

Higgs Mass in MSSM

- Higgs potential with soft terms

$$\begin{aligned} V &= (|\mu|^2 + m^2_{H_u})(|H^0_u|^2 + |H^+_u|^2) + (|\mu|^2 + m^2_{H_d})(|H^0_d|^2 + |H^-_d|^2) \\ &\quad + [b\,(H^+_u H^-_d - H^0_u H^0_d) + \text{c.c.}] \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H^0_u|^2 + |H^+_u|^2 - |H^0_d|^2 - |H^-_d|^2)^2 + \frac{1}{2}g^2|H^+_u H^{0*}_d + H^0_u H^{-*}_d|^2 \end{aligned}$$

- Minimize this potential with

$$v_u = \langle H^0_u \rangle, \qquad v_d = \langle H^0_d \rangle$$

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\ \text{GeV})^2$$

$$\tan\beta \equiv v_u/v_d$$

The μ Problem

- Correct EWSB then leads to

$$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu|^2$$

- Implies μ should be ~ SUSY breaking scale (unless cancellations)

- μ appears in Superpotential, so natural value is cutoff of the theory

- Why is it so small ?
  - Giudice-Masiero mechanism
  - NMSSM
  - Family Symmetry ?

The μ Term and Family Symmetry

- If the Higgs have family partners "μ" is now a matrix
- Diagonalization of the μ matrix leads to supersymmetric Higgs mass spectrum
- Family Symmetry could :
  - Forbid it explicitly (Ex : Z7 X Z3 with both Higgses being anti-triplets)
  - Allow it but give a hierarchy amongst the Higgses

The μ Term in $PSL_2(7)$

- Higgses are 6's under $PSL_2(7)$

$$\mathbf{6} \otimes \mathbf{6} = (\mathbf{1} + \mathbf{6} + \mathbf{6} + \mathbf{8})_s + (\mathbf{7} + \mathbf{8})_a$$

- μ term not forbidden by family symmetry
  - Could be forbidden by a "PQ-like" symmetry
  - Technical Naturalness in SUSY
- Want to use the Familons from the Majorana sector !

$$[\bar{\varphi}\bar{\varphi}']_{\mathbf{6}} + [\bar{\varphi}\bar{\varphi}']_{\mathbf{3}}$$

$$[\mathcal{H}_u\mathcal{H}_d]_{\mathbf{6}}[\bar{\varphi}\bar{\varphi}']_{\mathbf{6}}$$

A Simple Underlying Theory

- Try to mimic the Majorana sector

$$\mathcal{W}_{\text{Fam}} = \left([\bar{N}\,\bar{N}]_6 + f[\mathcal{H}_u\mathcal{H}_d]_{6_1}\right)\Phi + [\bar{\varphi}\bar{\varphi}']_6\bar{\Phi} + \Phi\bar{\Phi}S_7$$

- Generates the term

$$\frac{f}{M_\Phi}[\mathcal{H}_u\mathcal{H}_d]_{6_1}[\bar{\varphi}\bar{\varphi}']_6$$

- Will generate the μ matrix when $PSL_2(7)$ breaks and Familons acquire vev's
- Why 6_1?

A Hint of SU(3)?

$$\mathcal{PSL}_2(7): \quad 6 \otimes 6 = (1 + 6_1 + 6_2 + 8)_s + (7 + 8)_a.$$

- 6_1 is natural in SU(3)

$$SU(3): \quad \bar{6} \otimes \bar{6} = (\overline{15}' + 6)_s + \overline{15}_a$$

6_2 in $\mathbf{15}'$ (symmetric fourth rank tensor)

- Does it produce a suitable hierarchy?

A Suitable Hierarchy ?

- Want there to be one "light" eigenvalue
  - Question of scale
- Light eigenvalue should have the correct eigenvector
  - Basis fixed by getting the top-quark mass right

$$\boxed{\bar{H}_{u1} = v_u}$$

  - Light eigenvalue should be mostly along this direction

The μ matrix

$$\boxed{M_0' \mathcal{H}_u \, \boldsymbol{\mu} \, \mathcal{H}_d}$$

$$\begin{pmatrix} H_u & \bar{H}_u \end{pmatrix} \begin{pmatrix} r\lambda^{16} & -r\lambda^{12} & -r\lambda^{8} & 0 & -\sqrt{2}\lambda^4 & 0 \\ -r\lambda^{12} & \lambda^8 & \lambda^4 & 0 & 0 & \sqrt{2}r\lambda^8 \\ -r\lambda^8 & \lambda^4 & 1 & \sqrt{2}r\lambda^{12} & 0 & 0 \\ 0 & 0 & \sqrt{2}r\lambda^{12} & 0 & -\lambda^8 & -r\lambda^{16} \\ -\sqrt{2}\lambda^4 & 0 & 0 & -\lambda^8 & 0 & -1 \\ 0 & \sqrt{2}r\lambda^8 & 0 & -r\lambda^{16} & -1 & 0 \end{pmatrix} \begin{pmatrix} H_d \\ \bar{H}_d \end{pmatrix}$$

$$\bar{\varphi} \sim \begin{pmatrix} \bar{\alpha}\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}, \qquad \bar{\varphi}' \sim \begin{pmatrix} \bar{\alpha}'\lambda^8 \\ \lambda^4 \\ 1 \end{pmatrix}$$

$$M'_0 \sim \frac{f}{\sqrt{2}} \times 10^{14}\ \text{GeV}$$

Higgs masses

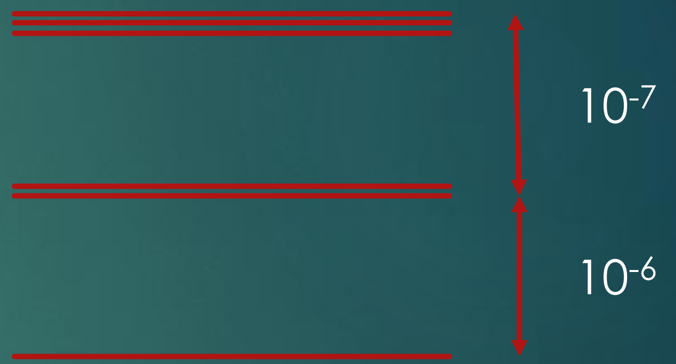
- Determined by :
  - Neutrino masses
  - Top quark hierarchy
  - f
  - CG coefficients

Spectrum

- Suitable spectrum ? Yes !

- Eigenvalues :

$$\sim \quad (-1, 1, 1, -2r\lambda^{12}, 2r\lambda^{12}, -8r^2\lambda^{24})$$
$$\sim \quad (-1, 1, 1, 10^{-7}, 10^{-7}, 10^{-13})$$



- Eigenvector of light eigenvalue in the right direction!

$$(.0035 \quad -0.000086 \quad 6.410^{-7} \quad -1.0 \quad -7.110^{-9} \quad -6.110^{-6})$$

Summary

- One light Higgs with eigenvalue : ~ 30 f GeV

- With SUSY breaking ~ TeV or not much higher, this will be the only Higgs to obtain a vacuum value

- Automatically aligned to give the right top quark mass!

$$h \;=\; \left(1+\mathcal{O}(\lambda^8)\right)\bar{H}_{u1}+\sqrt{2}\lambda^4\,H_{u1}+\cdots$$

Top Quark Yukawa's

- Not the full picture

$$m_t \begin{pmatrix} r(1-2r)\lambda^{16} & 3r\lambda^{12} & -r\lambda^{8} \\ 3r\lambda^{12} & -\lambda^{8} & \lambda^{4} \\ -r\lambda^{8} & \lambda^{4} & -1 \end{pmatrix}$$

$$m_t, \qquad 2r\lambda^{12} m_t, \qquad -2r\lambda^{12} m_t$$

- Up and charm quark masses too small

Not a complete Model!

- Mechanism to generate light quark masses
  - Radiatively? (loops) $\boxed{(1/16\pi^2 \sim 2\lambda^4)}$
  - New degrees of freedom?
  - Must be tied in with SUSY breaking
- Must pick the right six
  - Picking the other does not gives different spectrum
- Model contains 3 extra U(1)'s
  - A new PQ like symmetry forbids singlet μ term
- Mysterious f $\boxed{E_6? \quad \left([\bar{N}\,\bar{N}]_6 + f[\mathcal{H}_u\mathcal{H}_d]_{6_1}\right)}$

Conclusions

- Family symmetry an interesting route to physics of Majorana mass and μ term
- Special Majorana Matrix seems to be natural in $PSL_2(7)$
- Hierarchy from the top quark sector can be successfully followed to Majorana and Higgs sectors
- Simple $PSL_2(7)$ underlying theories
- Proof of concept, more to come!